



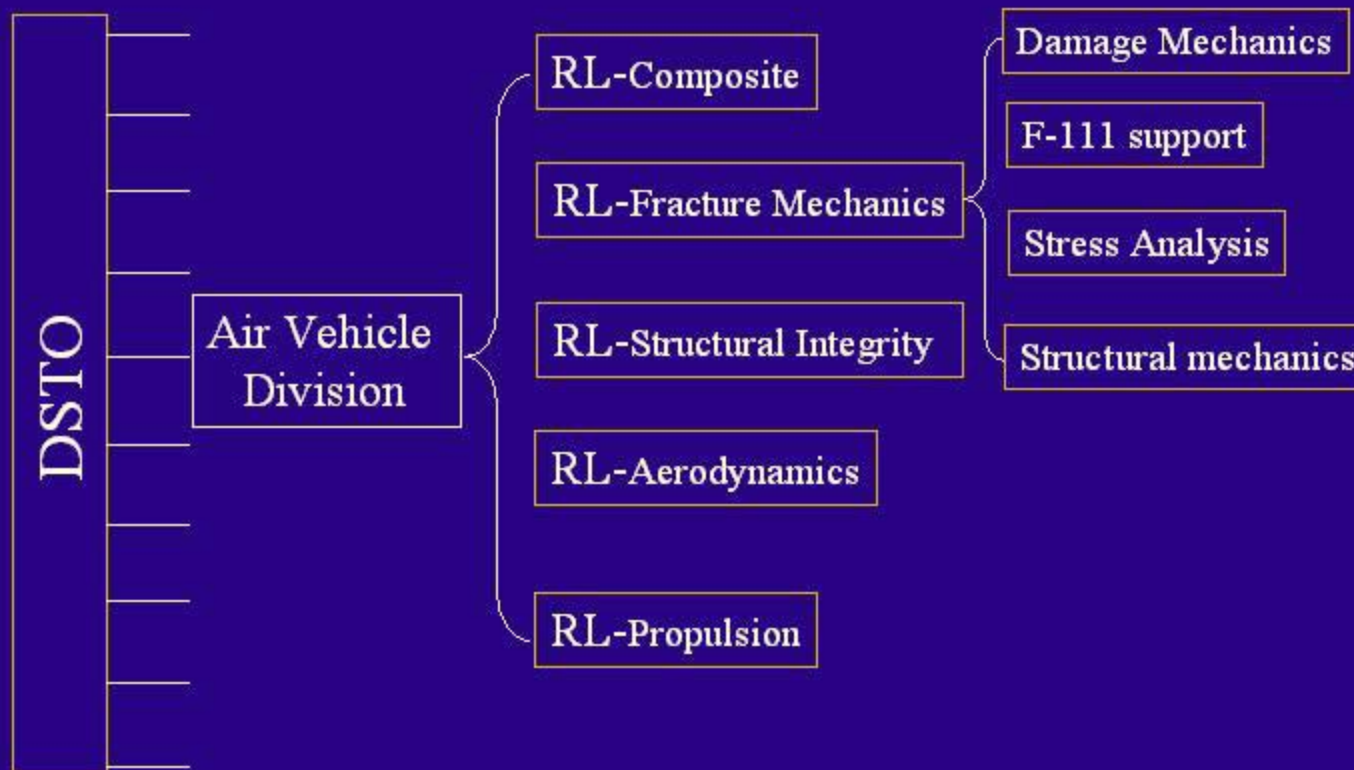
# **Recent Progress in the Development of Predictive Capability for Small Fatigue Crack Growth**

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Melbourne, Australia**



## Defence Science and Technology Organisation





## **Overview**

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- ☐ **Motivation and Context.**
- ☐ **Some Experimental Observations.**
- ☐ **Theoretical Considerations.**
  - **Correlating parameter;**
  - **Plasticity induced crack closure;**
- ☐ **Computational approach.**
  - **Grossly plastic deformation;**
  - **Strain gradient (notch plastic zone).**
- ☐ **Applications.**
  - **Crack growth through residual stress field.**
- ☐ **Conclusions.**

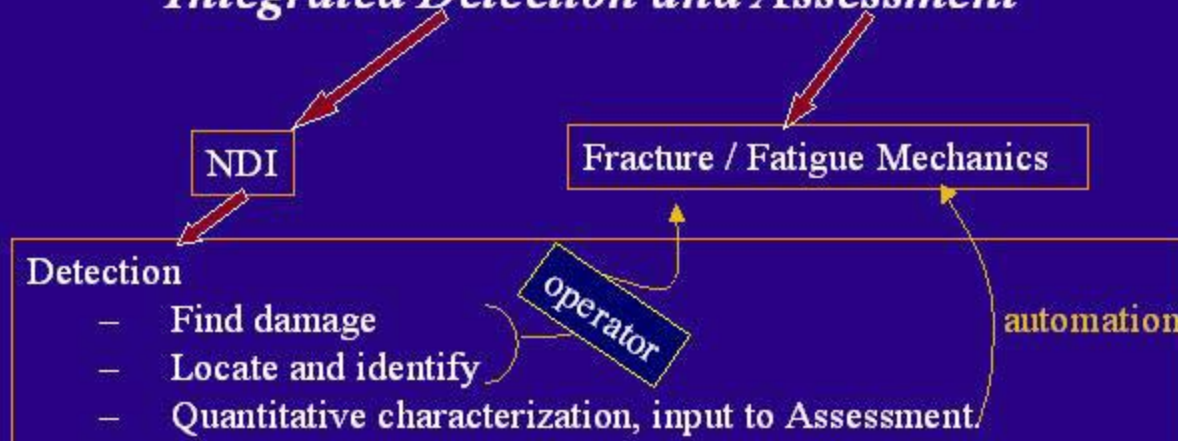


## Motivation and Context

Improvement over current engineering  
practice in life prediction



**Prognosis Health Management**  
*Integrated Detection and Assessment*





## **Motivation and Context**

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### **A brief history:**

1. 1955: First paper on fatigue crack growth using dislocation theory by Dr. Alan Head.
2. 1995: Long crack well understood and predictive tools (FASTRAN by Jim Newman).
3. 2015: Adaptive, 3D computational crack growth?

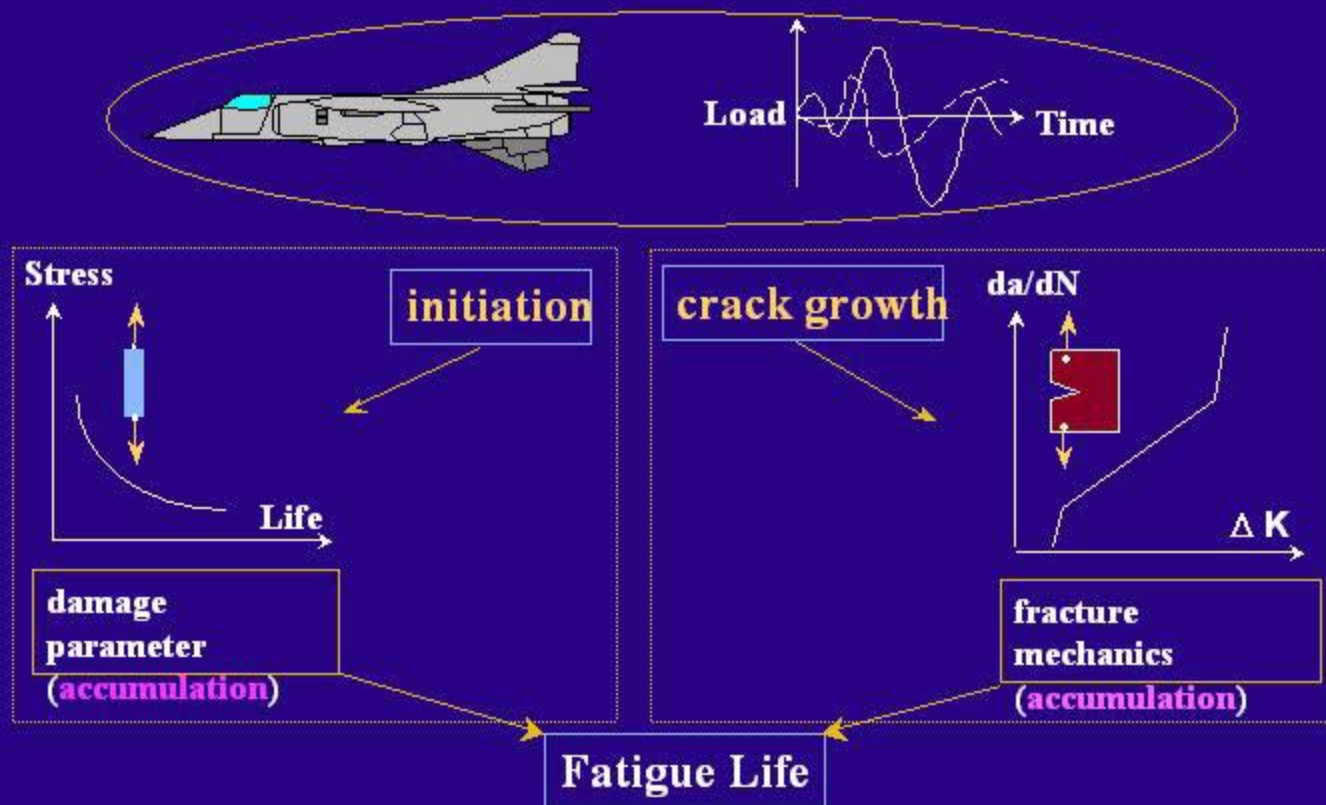
### **Challenges:**

1. Correlating parameter for small cracks ( $\geq 10 \mu\text{m}$ ).
2. Crack closure (plasticity and surface roughness induced).
3. Scale effect (crack size relative to microstructure dimension).



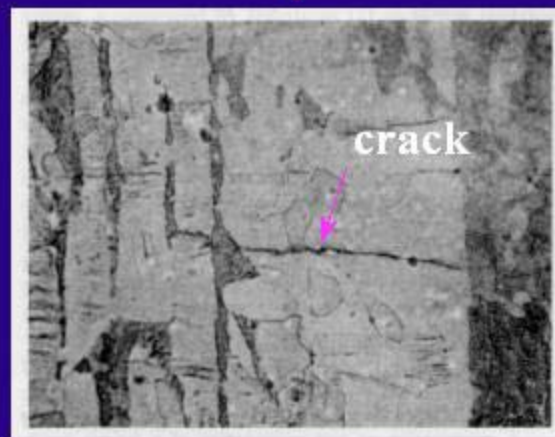
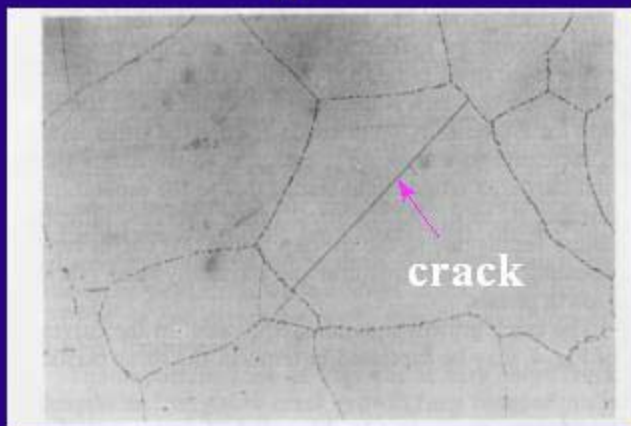


## "fatigue crack growth modelling"





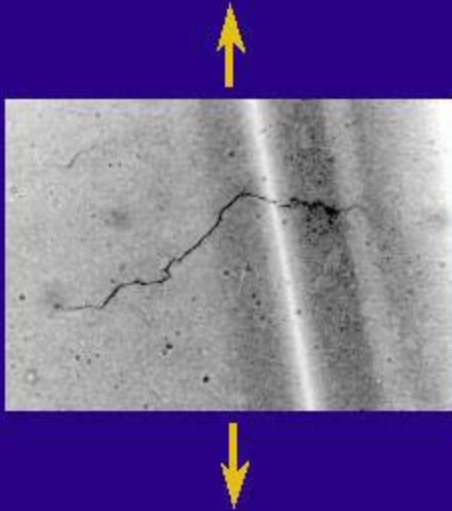
## Some Experimental Observations



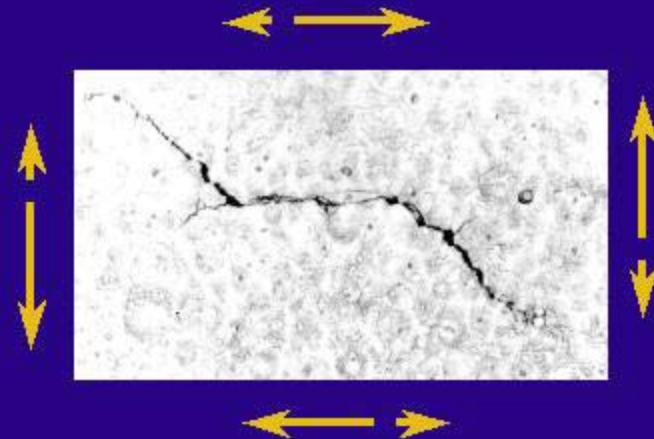


## Some Experimental Observations

**tension**



**torsion**

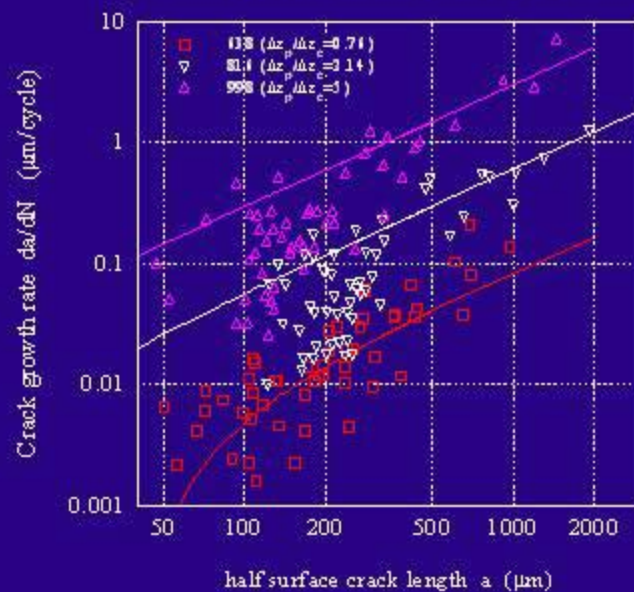




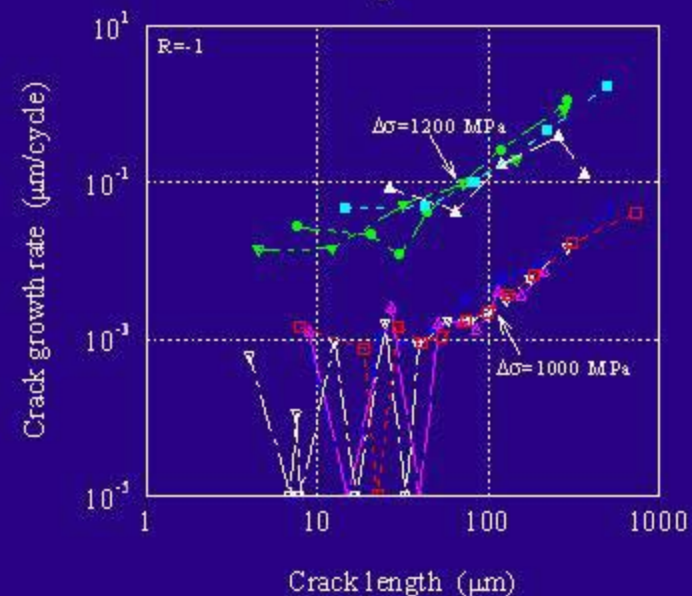


## Some Experimental Observations

Mild carbon steel



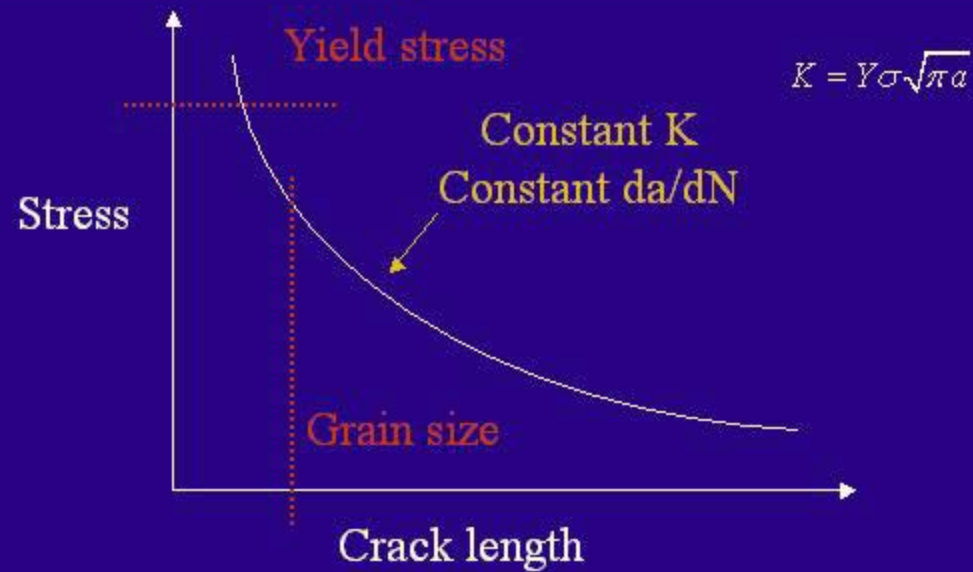
Alloy



Failure of Stress Intensity Factor to Correlate Crack Growth Rates



## Correlating Parameter

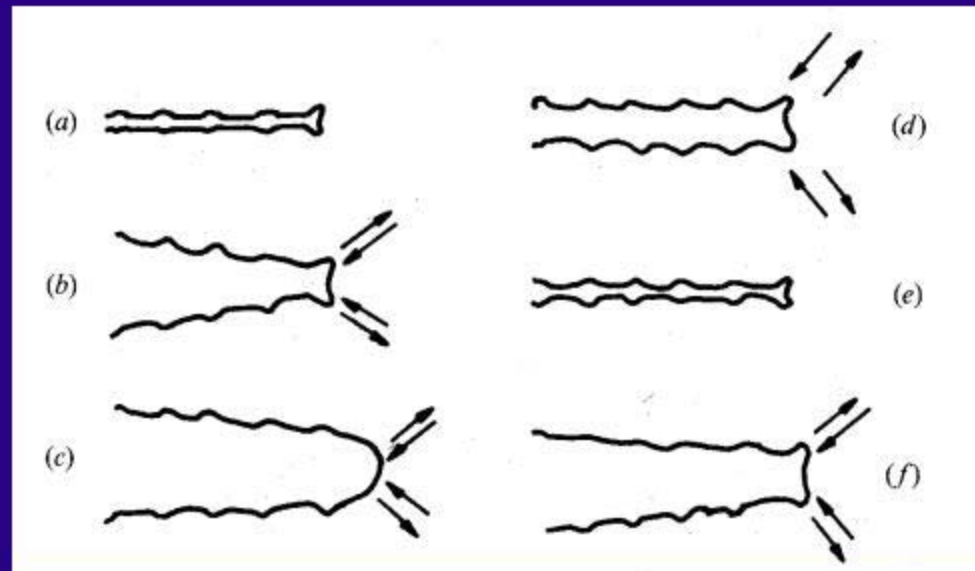


- (i) Failure of similitude
- (ii) Failure of continuum hypothesis



## Correlating Parameter

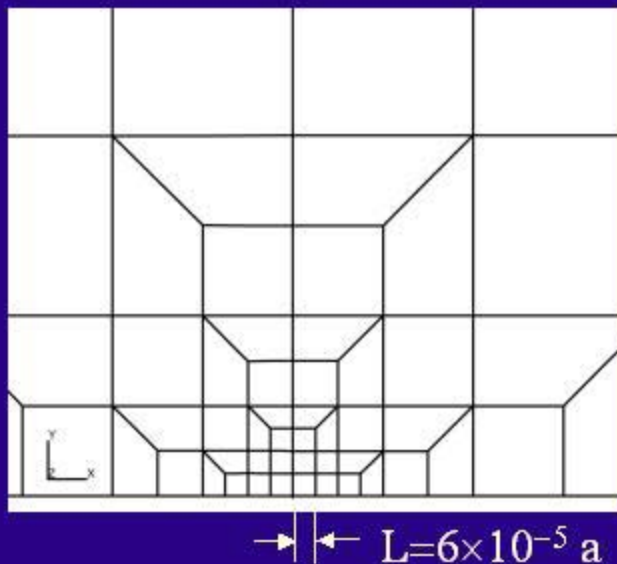
Stage II growth by plastic blunting at crack tip (Laird, 1967)



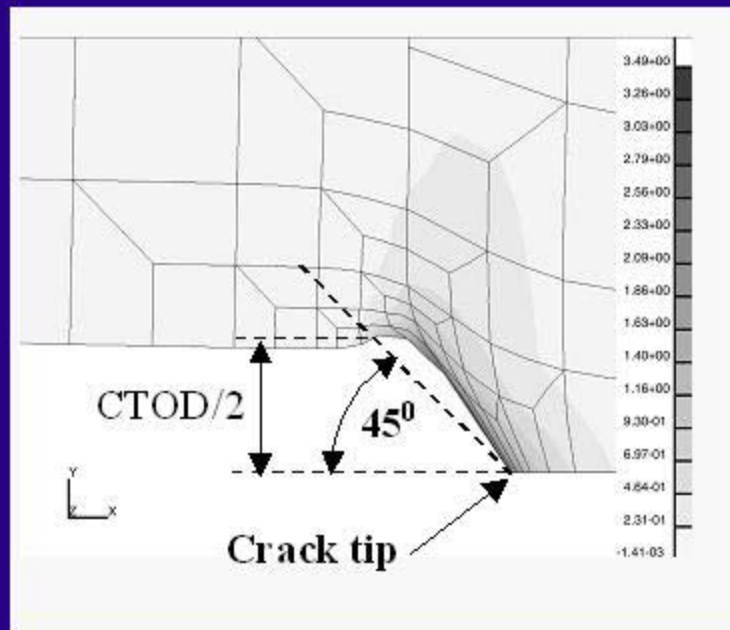


## Correlating Parameter for Small Cracks

Un-deformed geometry



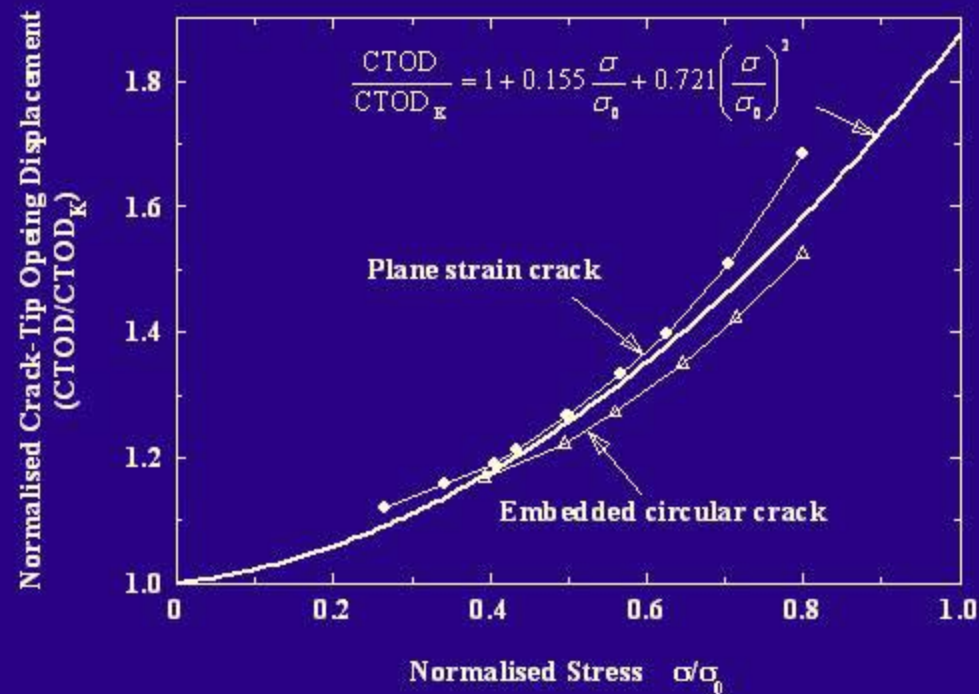
Deformed geometry





## Correlating Parameter for Small Cracks

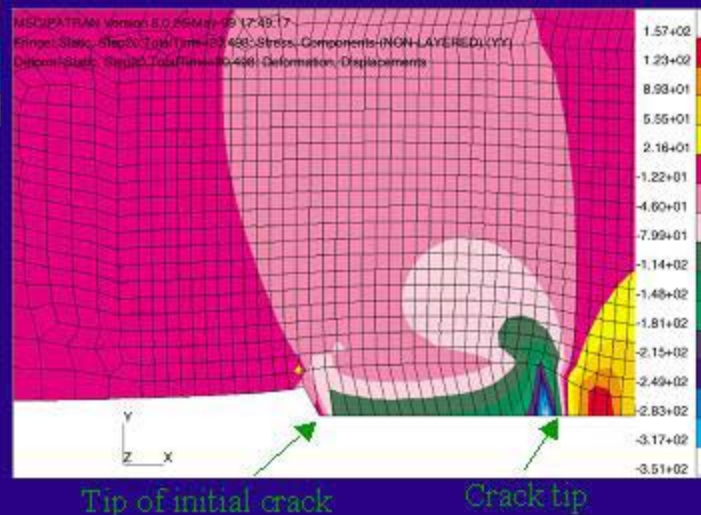
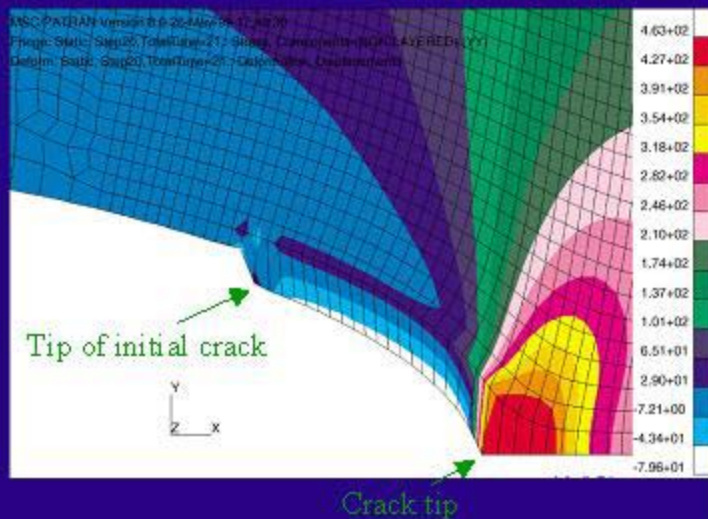
### Cyclic Crack-tip Plastic Bunting







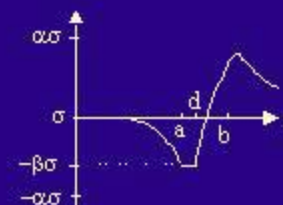
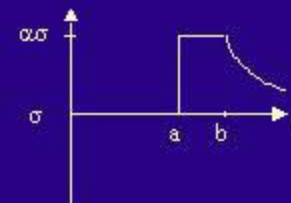
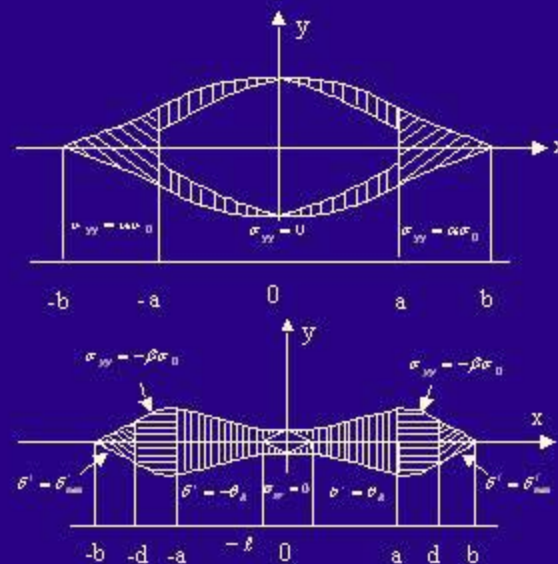
## Crack Closure under Cyclic Loading





## Modified Crack Closure Solution

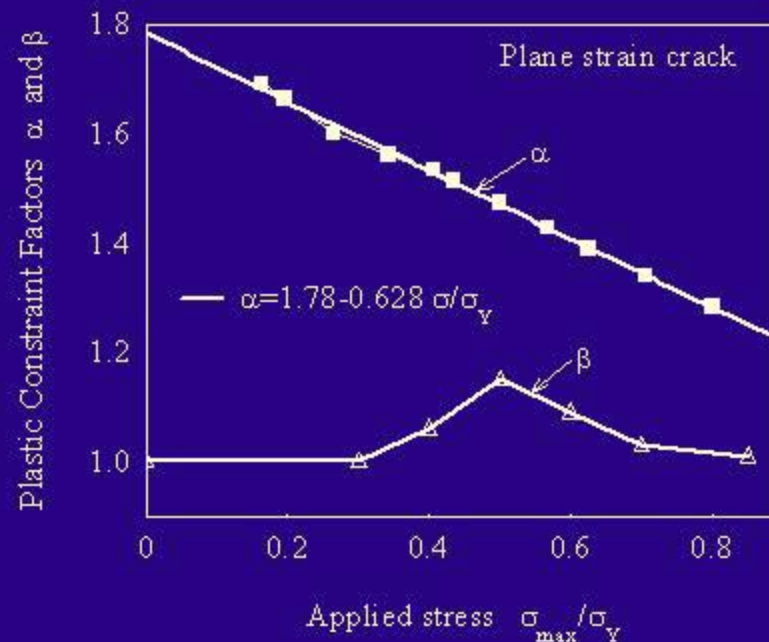
- ❑ Simulating three-dimensional plastic deformation near crack tip using two-dimensional Dugdale model.
- ❑ Two plastic constraint factors.





## Modified Crack Closure Solution

- Two plastic constraint factors: determined by matching analytical and FE solutions.
- Input to modified FASTRAN for spectrum loading.

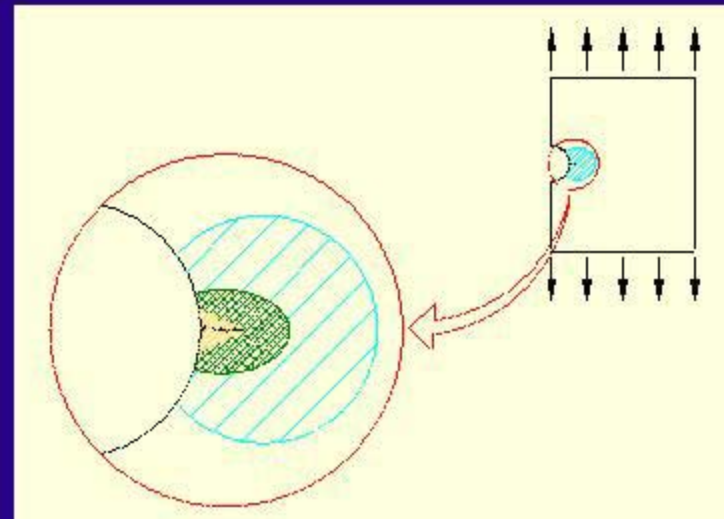




## Strain Gradient Effect

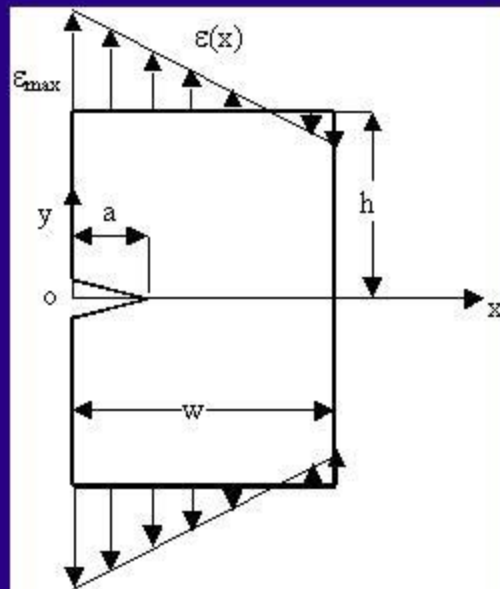
### ❑ Small crack emanating from notch plastic zone

- Cyclic Notch Plasticity
- Small Cracks
  - Non-SSY
  - Plastic Strain Controlled
  - Microstructure

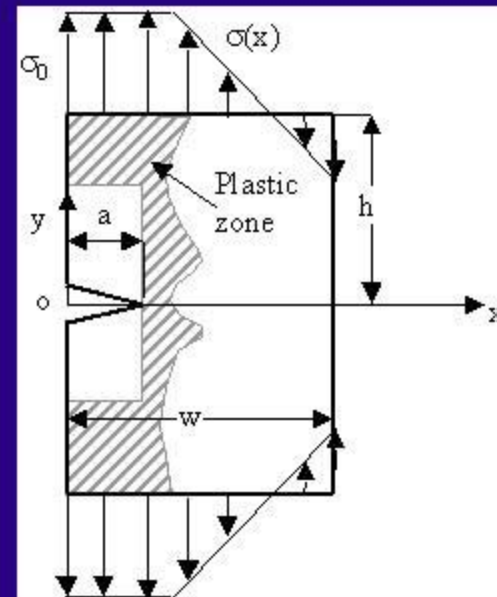




## Strain Gradient Effect



Applied Strain



Elasto-plastic Stress

Stress-based Dugdale Models I & II

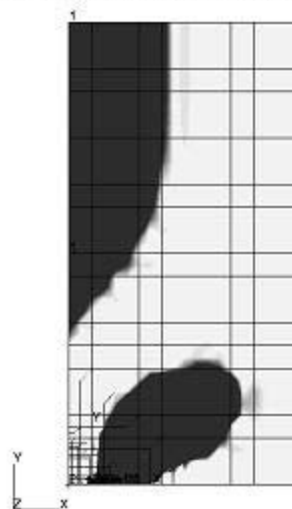




## Strain Gradient Effect

Front face yielding

MSOPATRAN Version 6.5.00-Aug-99 13:41:55  
Fringe: Default, Step1, TotalTime=0.39008, Plastic Strain, Components-(NON-LAYERED) (YY)



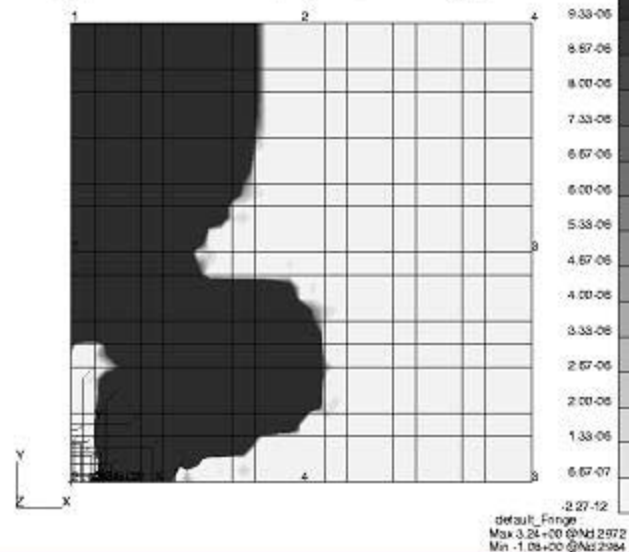
Break-through

MSOPATRAN Version 6.5.00-Aug-99 13:41:15  
Fringe: Default, Step1, TotalTime=0.44359, Plastic Strain, Components-(NON-LAYERED) (YY)



Gross yielding

MSOPATRAN Version 6.5.00-Aug-99 13:42:32  
Fringe: Default, Step1, TotalTime=0.63016, Plastic Strain, Components-(NON-LAYERED) (YY)





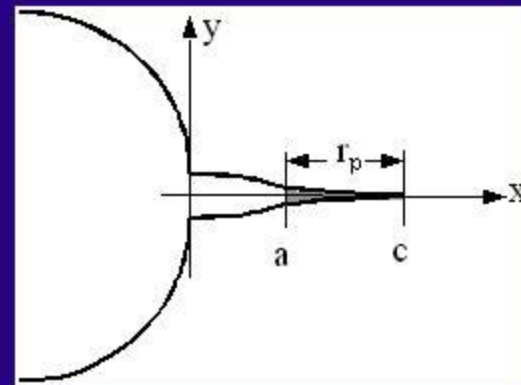
## Strain Gradient Effect

### Strain-based Dugdale Model

$$\int_0^{c_3} D(t) Q(x, t) dt = \begin{cases} -\sigma_{yy}(x) / E' & (0 < x < a) \\ [-\sigma_{yy}(x) + \sigma_0] / E' & (a < x < c_3) \end{cases}$$

$$\int_0^{c_3} D(t) Q(x, t) dt = \begin{cases} -\varepsilon_{ep}(x) & (0 < x < a) \\ -\varepsilon_{ep}(x) + \sigma_0 / E_s(x) & (a < x < c_3) \end{cases}$$

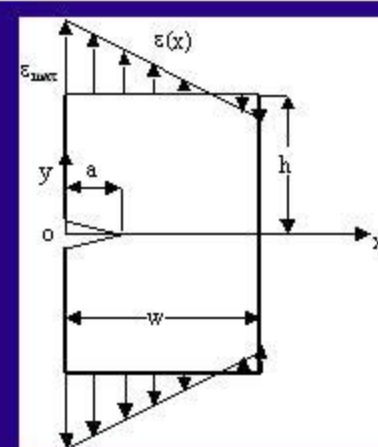
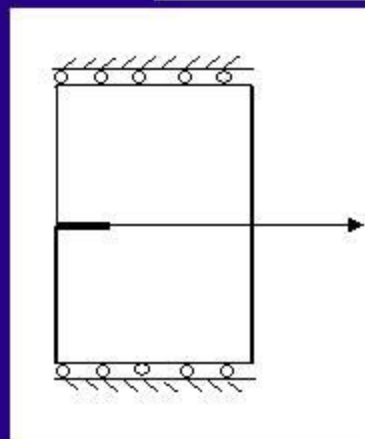
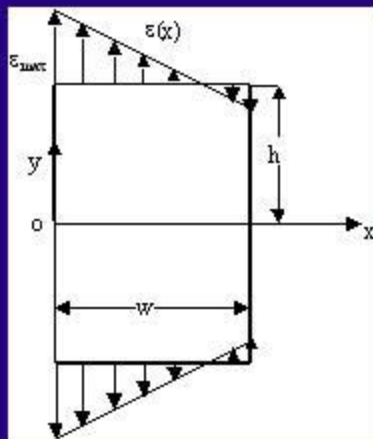
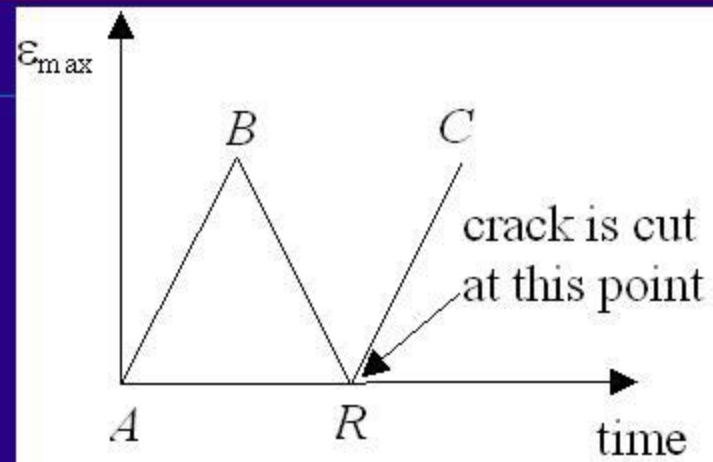
$$E_s(x) = \sigma_{ep}(x) / \varepsilon_{ep}(x)$$





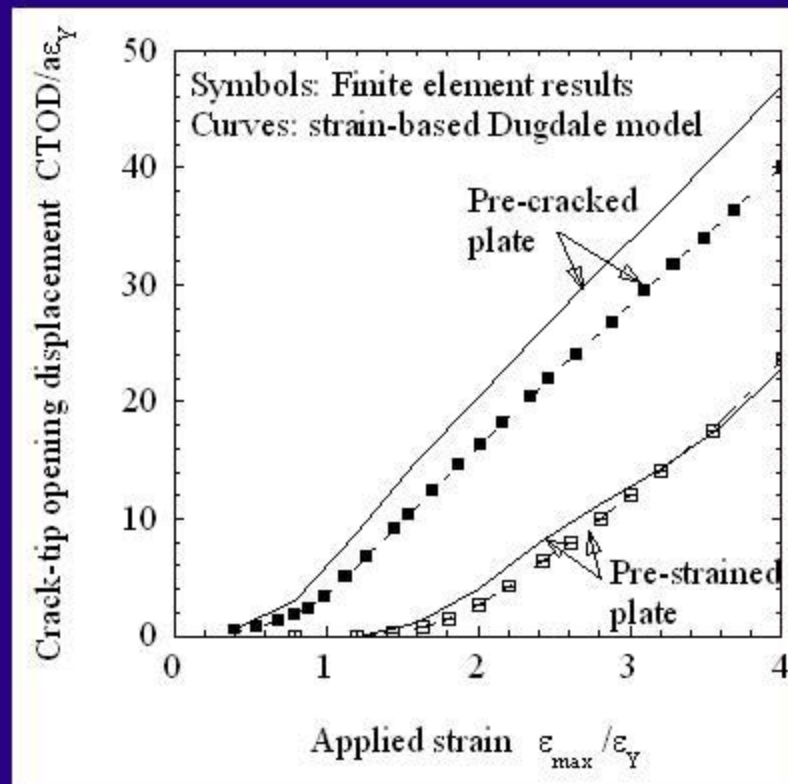
## Strain Gradient Effect

Load history of pre-straining





## Strain Gradient Effect





## **Crack Growth Through Residual Stress Field**

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### **Surface enhancement techniques**

- **Shot peening**
- **Laser shock**
- **Low plasticity burnishing**

### **No credit is taken of the benefit of residual stresses**

- **Quality assurance.**
- **Stability of residual stresses under mechanical or thermal loading.**
- **Fatigue crack growth: influence of flaws.**





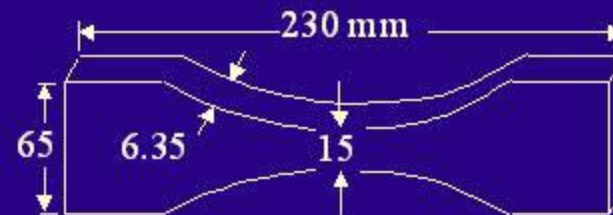
## Experiments

### Aluminum alloy (7050)

- ☐ Etched
- ☐ Shot peened

### Loading

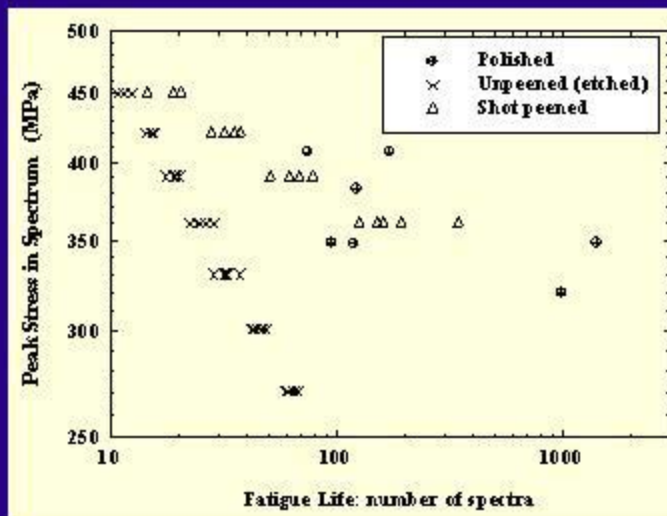
- ☐ FALSTAFF
- ☐ Modified sequence representative of F/A-18



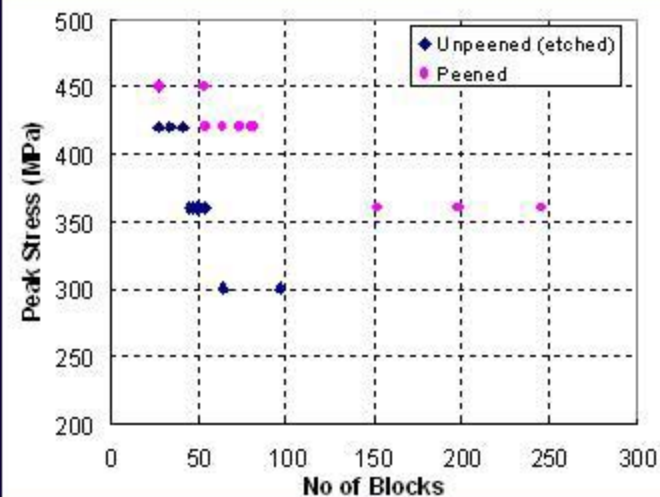


## Experimental Results

F/A-18 sequence

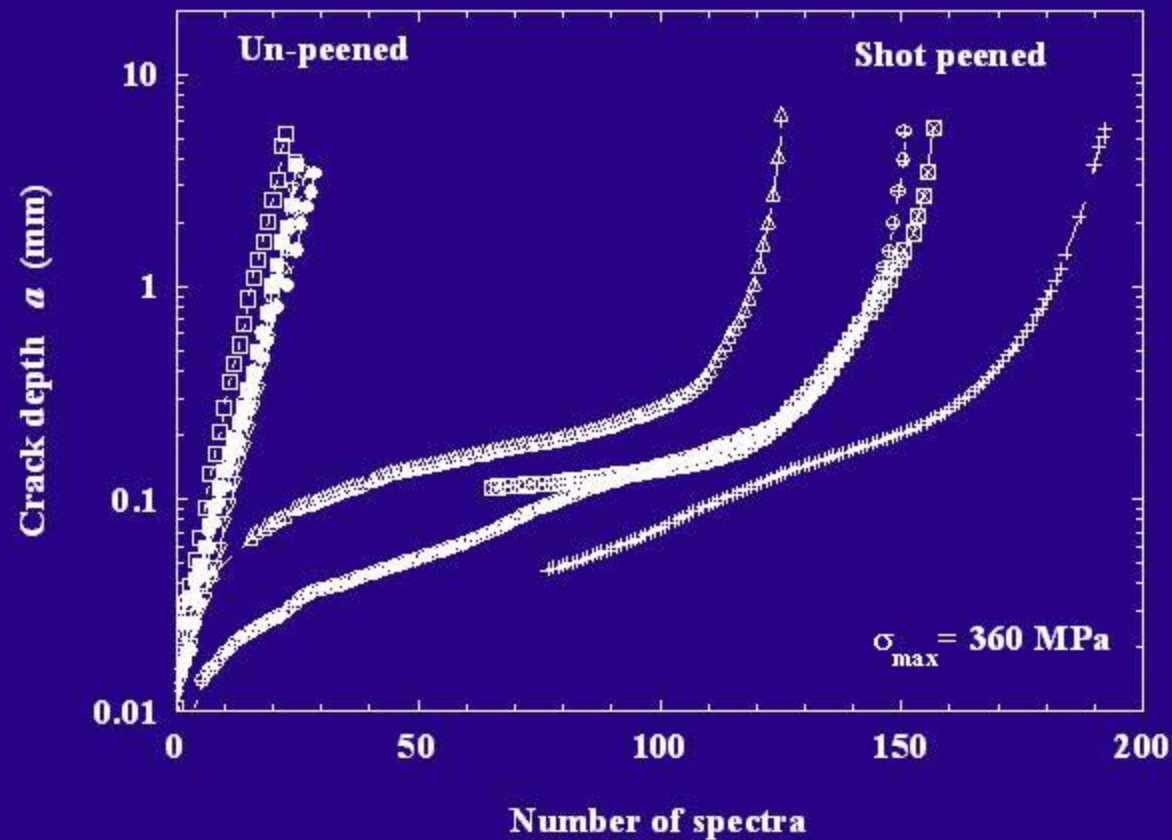


FALSTAFF



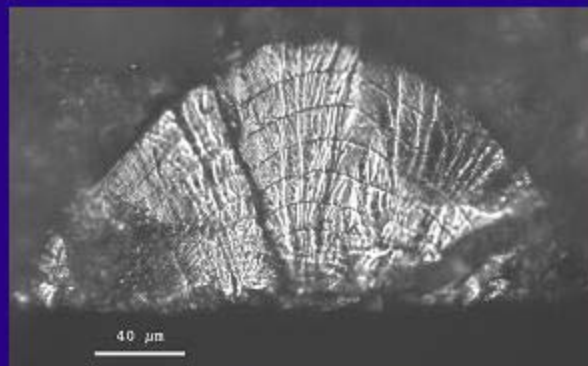


## Crack Growth Results



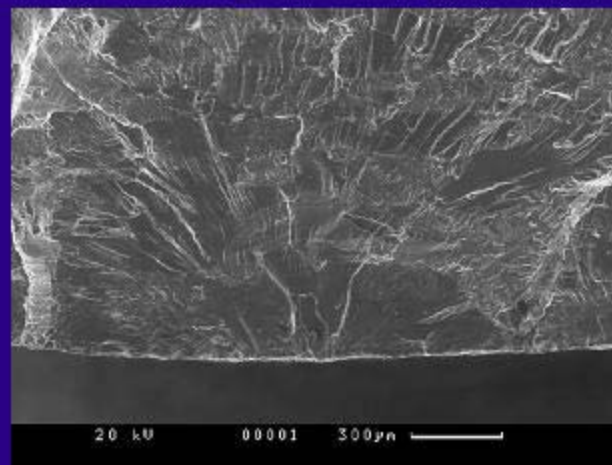
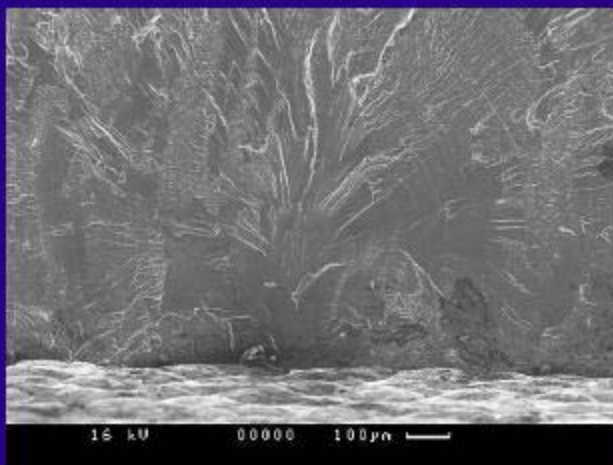


## Fractographic Results



Un-peened

Shot peened





## Fatigue Crack Growth Prediction

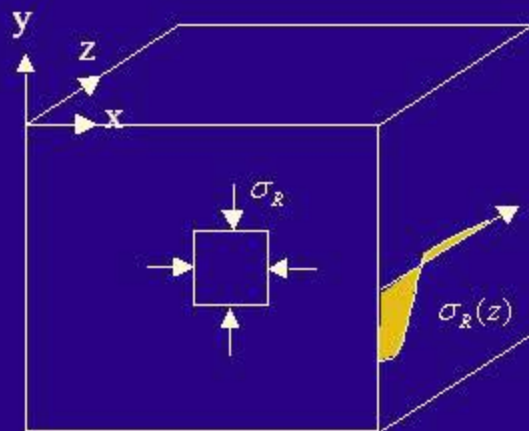
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- ❑ Correlating parameter;
  - Cyclic crack-tip opening displacement.
- ❑ Cycle-by-cycle crack closure analysis;
  - Effect of residual stress.
  - Modified Dugdale model (e.g., FASTRAN).
- ❑ Material database;
  - $da/dN$  .vs. correlating parameter (threshold for small cracks).
- ❑ Initial flaw size;
  - Fractographic analysis.

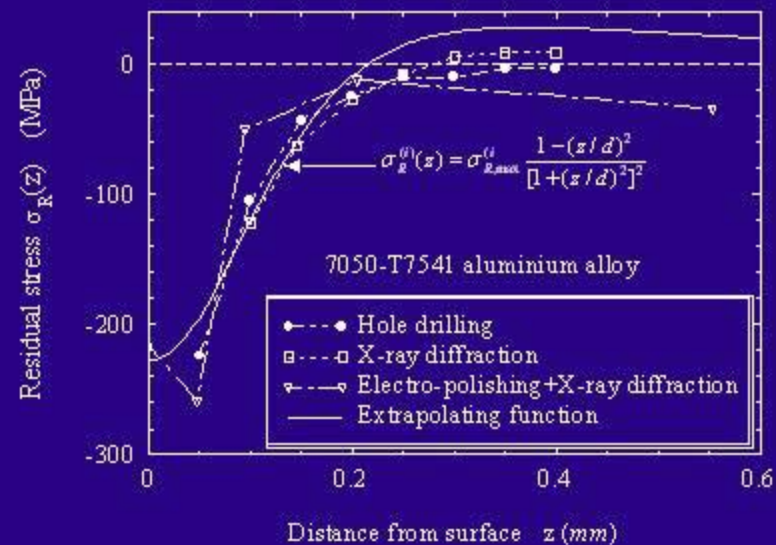




## RESIDUAL STRESS



$$\sigma_R^{(i)}(z) = \sigma_{R,max}^{(i)} \frac{1 - (z/d)^2}{[1 + (z/d)^2]^2}$$



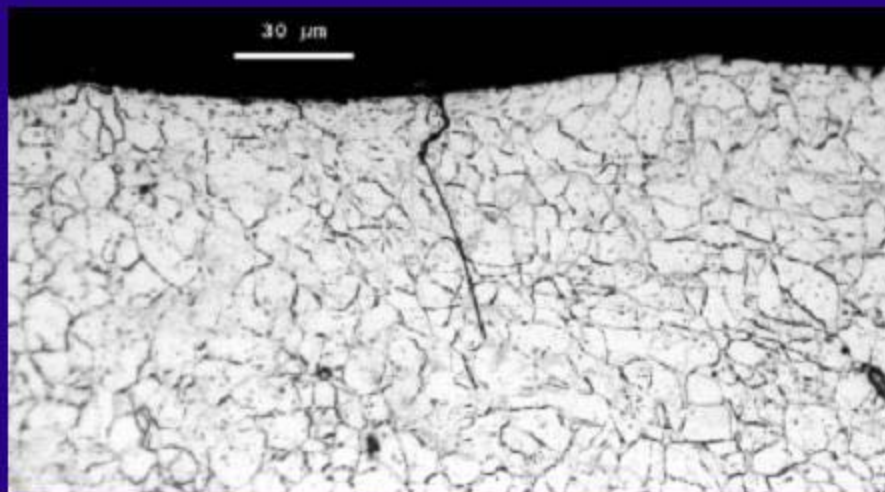
$$\int_0^\infty \sigma_R^{(i)}(z) dz = 0$$



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**Air Vehicle Division**

## **Initial Flaws due to Shot Peening**





## **Stability of Residual Stresses**

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### **Stress shake-down:**

- **Elastic shake-down: zero alternating plasticity.**
- **Plastic shake-down: alternating plasticity (zero mean stress).**

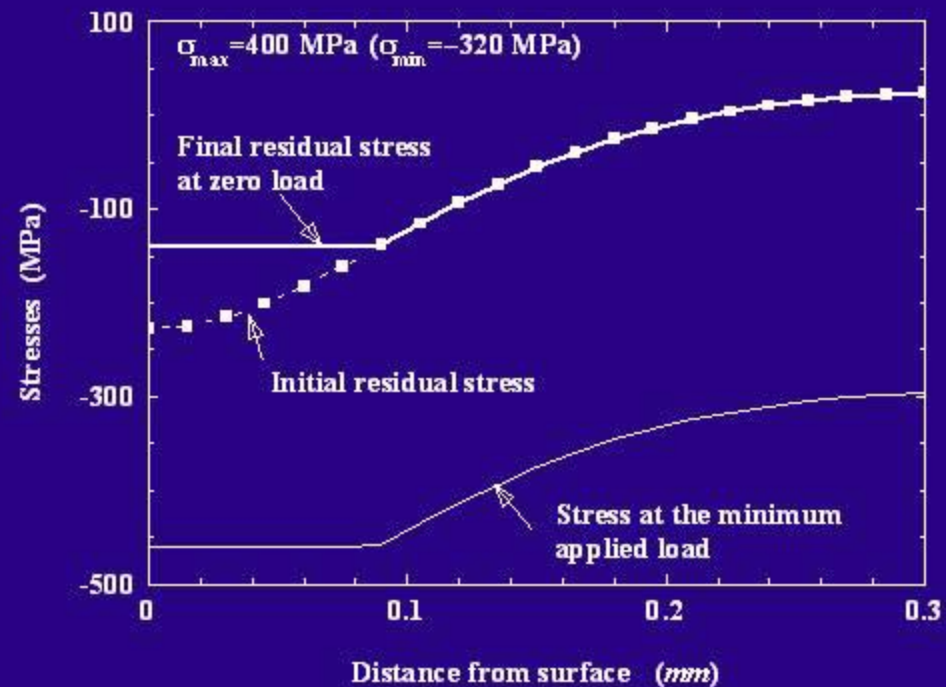
### **Compressive residual stress**

- **High compressive load may cause elastic or plastic shake-down.**
- **Rate of shake-down depends on material properties and applied loads.**



## Relaxation of Residual Stress

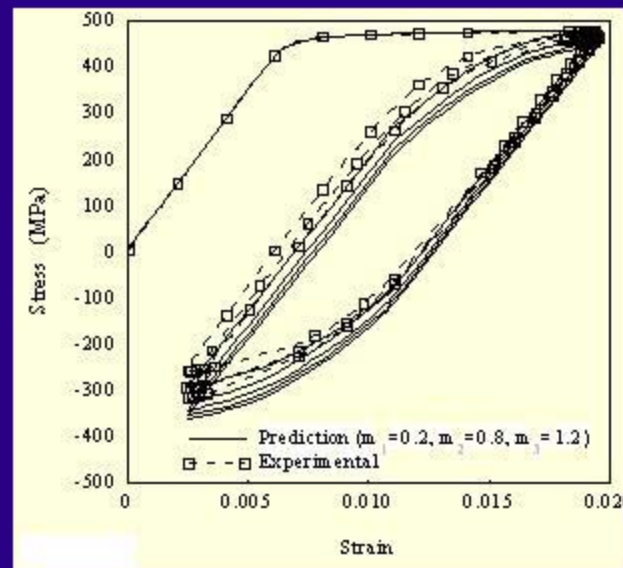
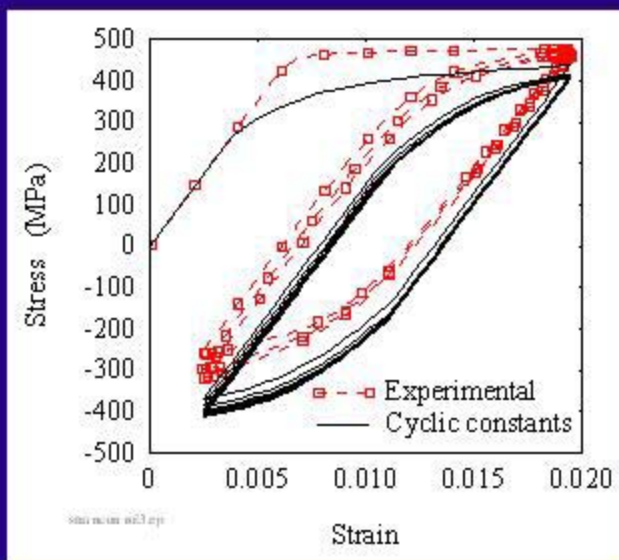
Perfectly plastic  
material





## Cyclic Deformation

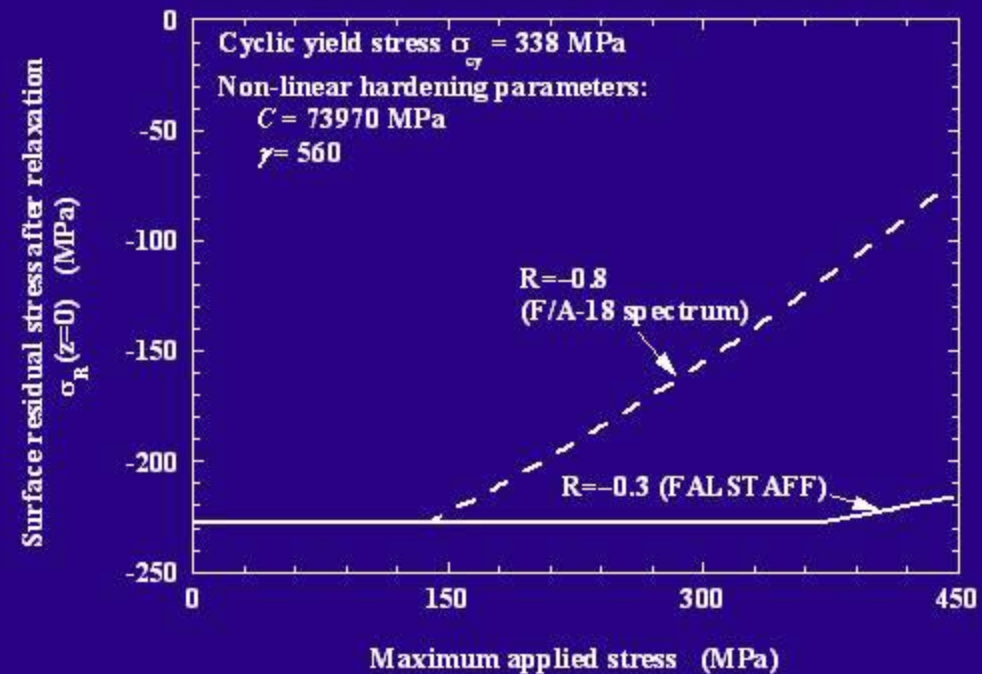
### 7050 Aluminium





## Relaxation of Residual Stress

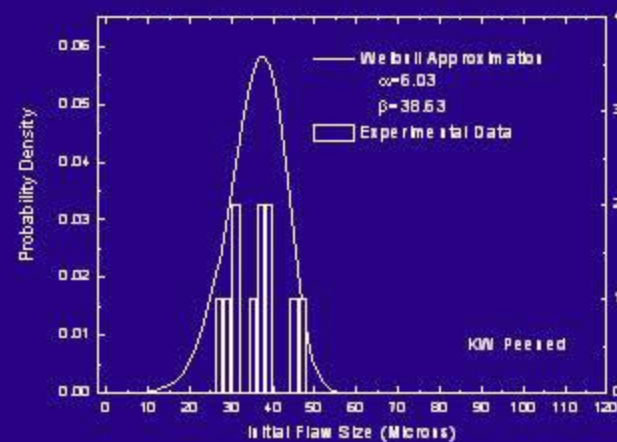
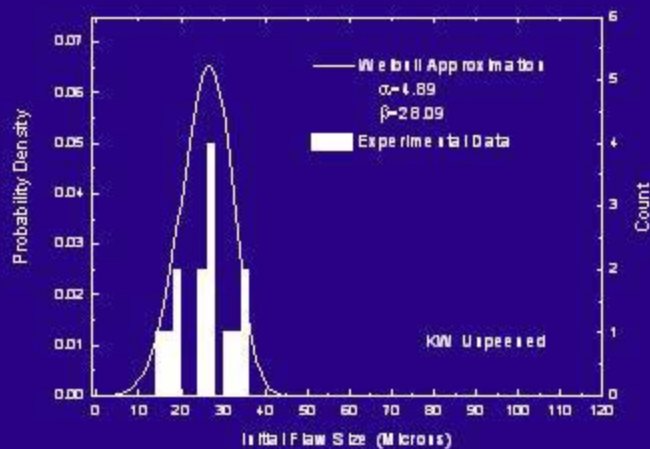
Non-linear  
Kinematic hardening







## Distribution of Initial Flaws



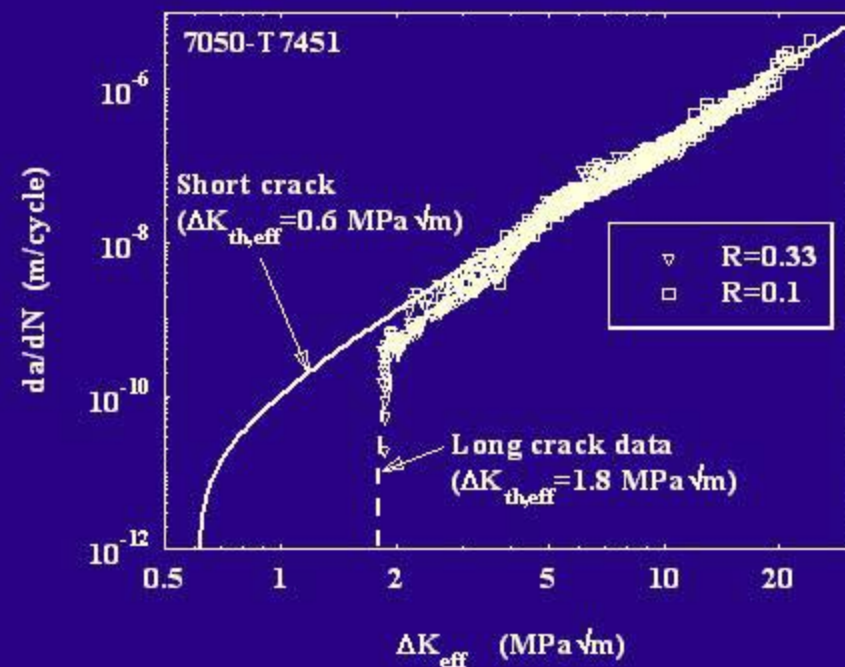


## Material Database

Threshold value is determined so that the initial flaw does not propagate at fatigue limit.

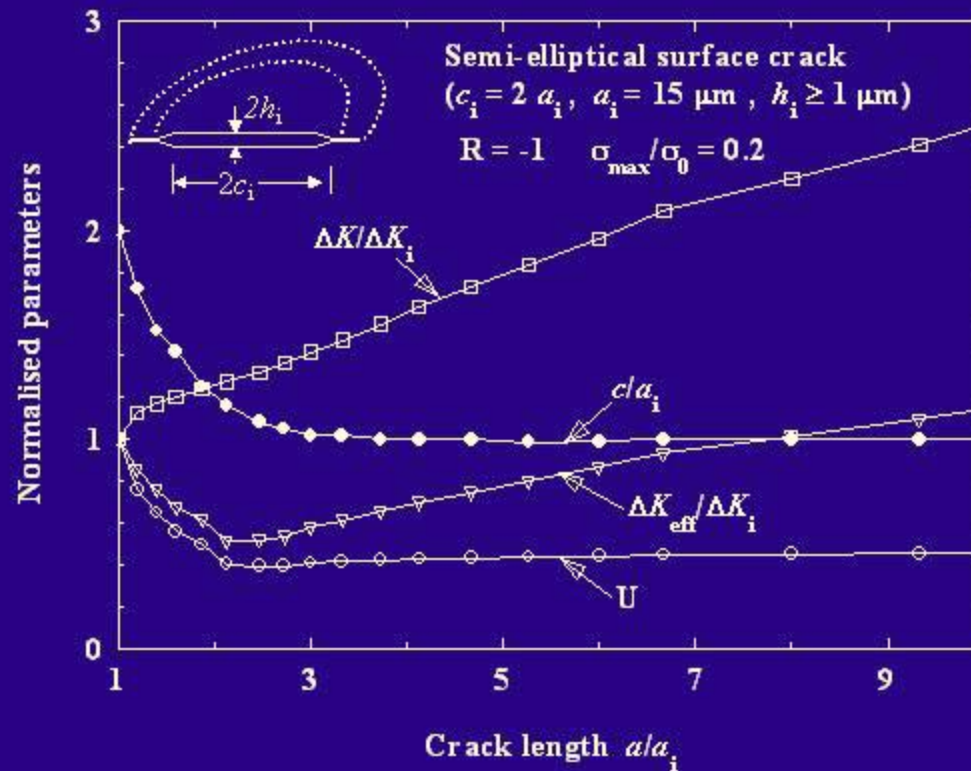
Input:

- (1) Flaw size
- (2) Fatigue limit





## Threshold Determination



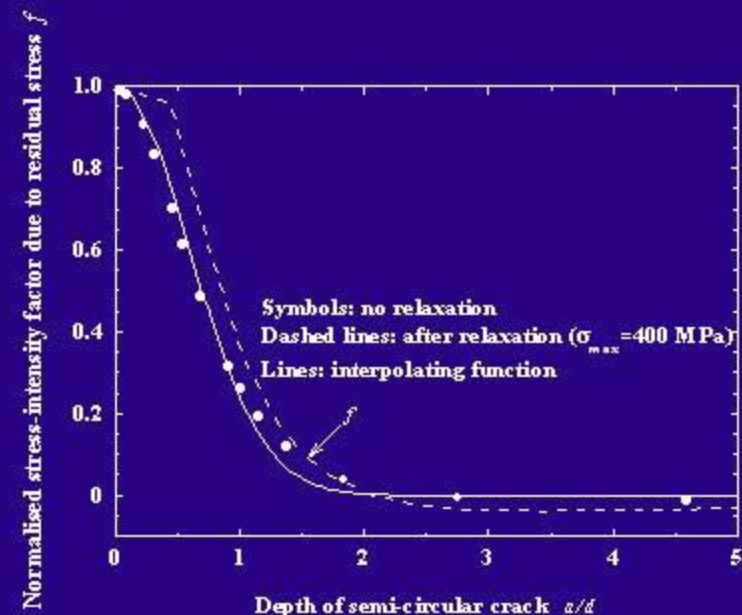
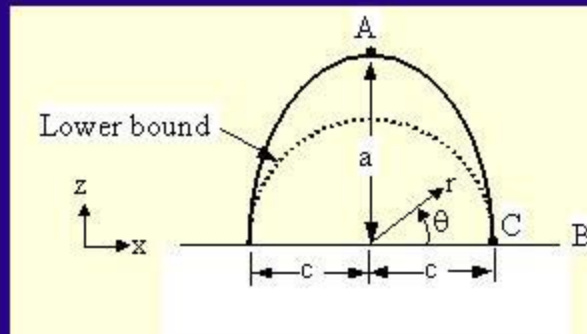


## Influence of Residual Stress on Crack Growth

$$K^R = \int_0^a \int_0^\pi \sigma_R(r \sin \theta) G(r, \theta) r d\theta dr$$

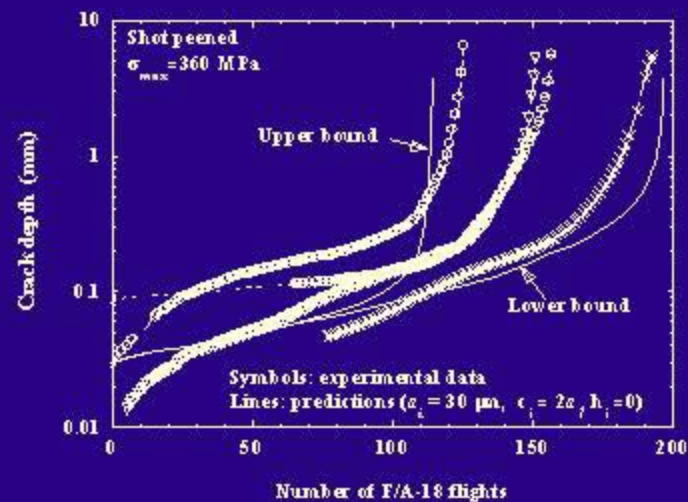
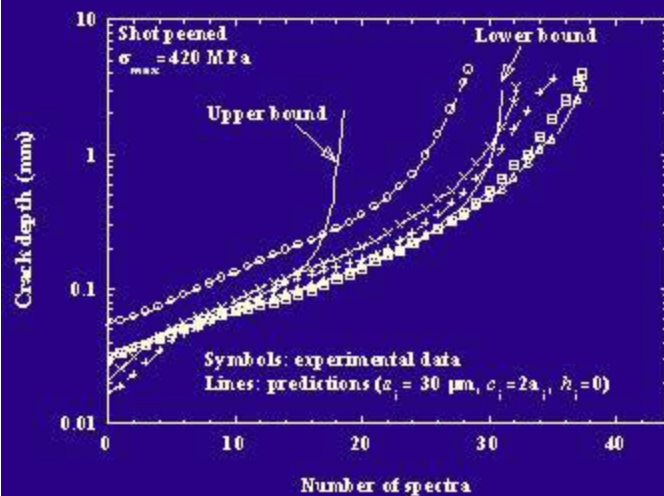
$$G(r, \theta) = \frac{\sqrt{a^2 - r^2}}{\pi \sqrt{\pi a}} \left[ \frac{1}{a^2 + r^2 - 2ar \sin \theta} + \frac{1}{a^2 + r^2 + 2ar \sin \theta} \right]$$

$$f = \frac{K^R}{2\sigma_{Rmax} \sqrt{a/\pi}}$$



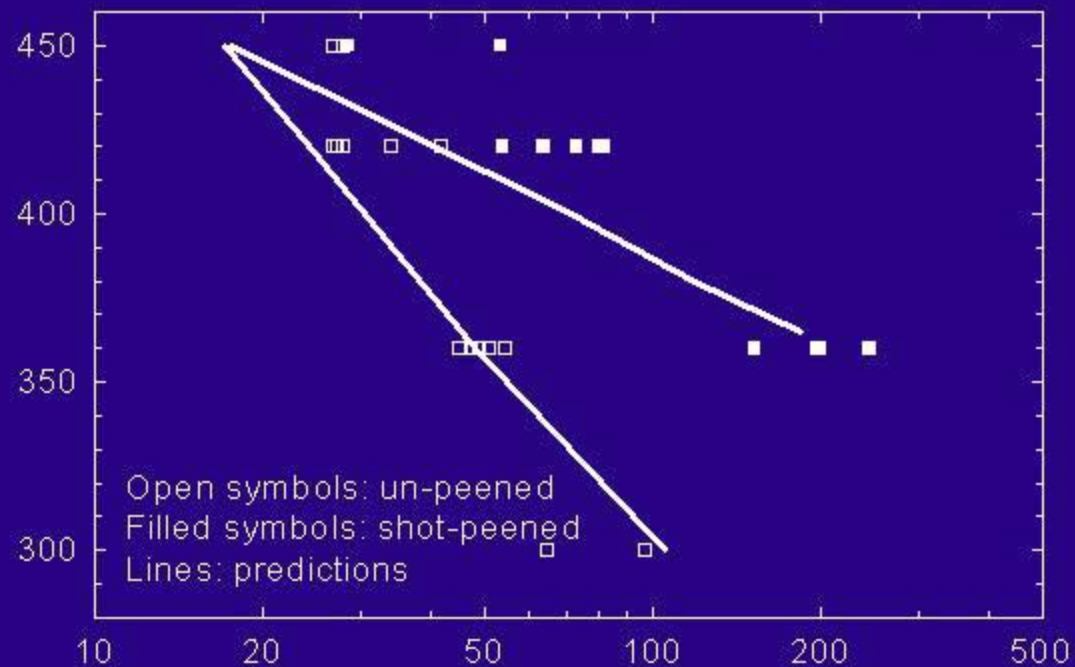


## Crack Growth Curves





## Comparison of Prediction with Experimental Data







## **Summary**

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- ☐ **Correlating parameter for short crack growth.**
- ☐ **Strain-based cohesive zone model for strain-controlled crack growth at notch root.**
- ☐ **Modified crack closure model for large-scale yielding.**
- ☐ **Analytical method for characterizing the relaxation rate and the final distribution of residual stresses.**
- ☐ **Predictive tools for assessing effect of surface enhancement.**